NetOpt 2022

Enumération des stratégies optimales des opérateurs de réseaux mobile

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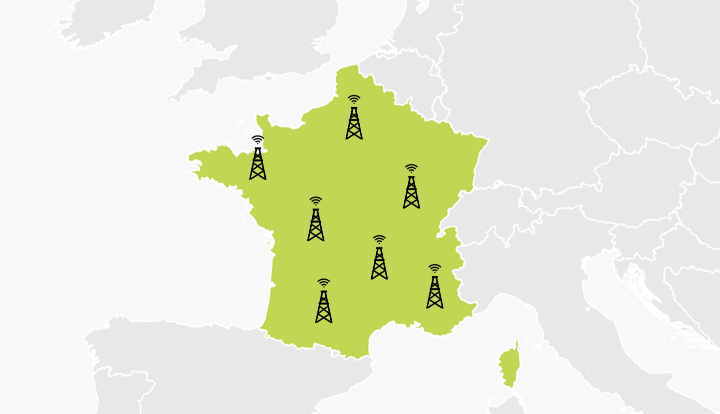
Contents

Topic

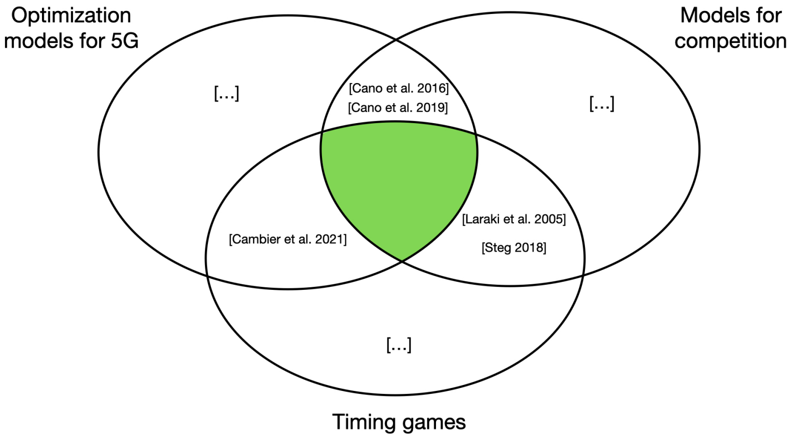
The telecom market is highly competitive. Every year operators invest 2-3 billion euros in their network in France.

How do operators prioritise the investments in light of competition?



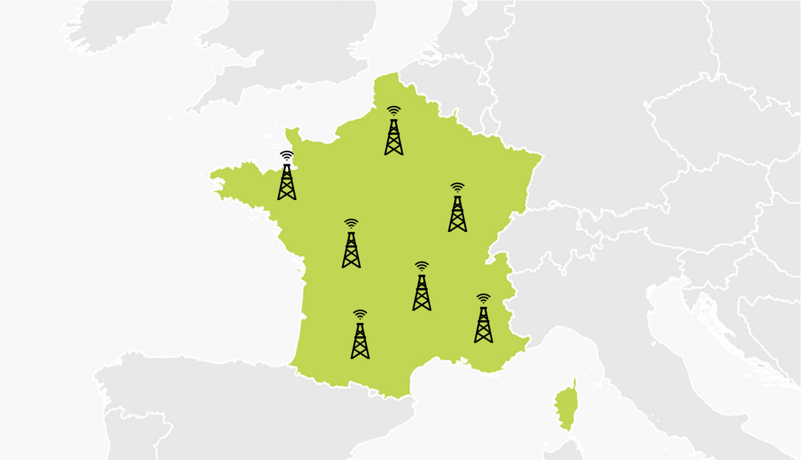
Literature

Our work presents an optimization model for the deployment of 5G which exploits results from game theory.



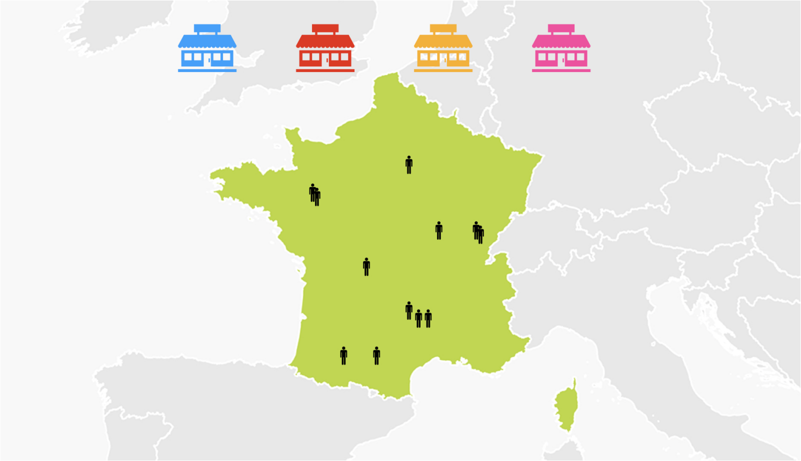
Two-phase model

We model the adoption of a new technology on a two-phase model [Zappalà et al. 2022].



Two-phase model: Phase 1

The operators choose independently the budget for the promotion of the technology.



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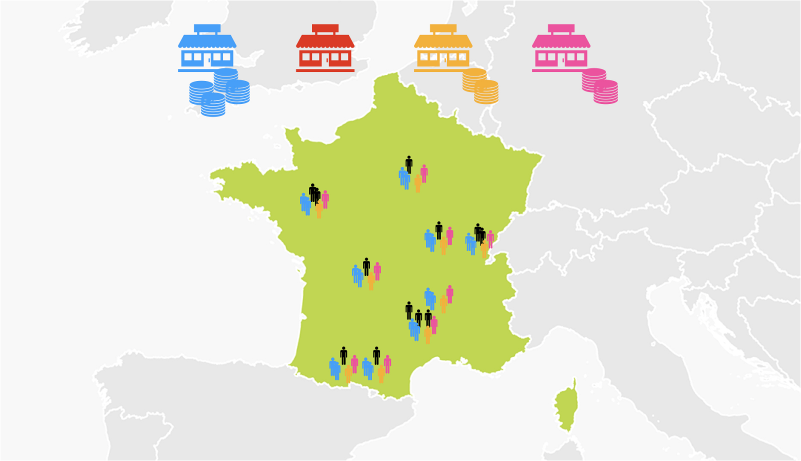
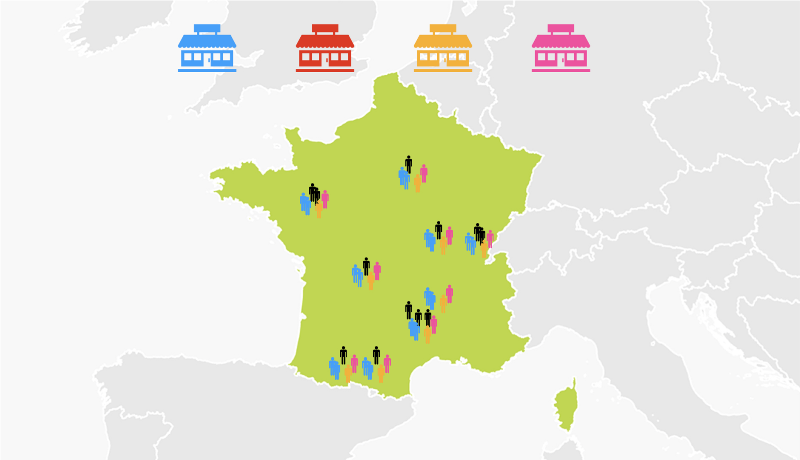


Two-phase model: Phase 2

The operators update the network observing the actions of the competitors.



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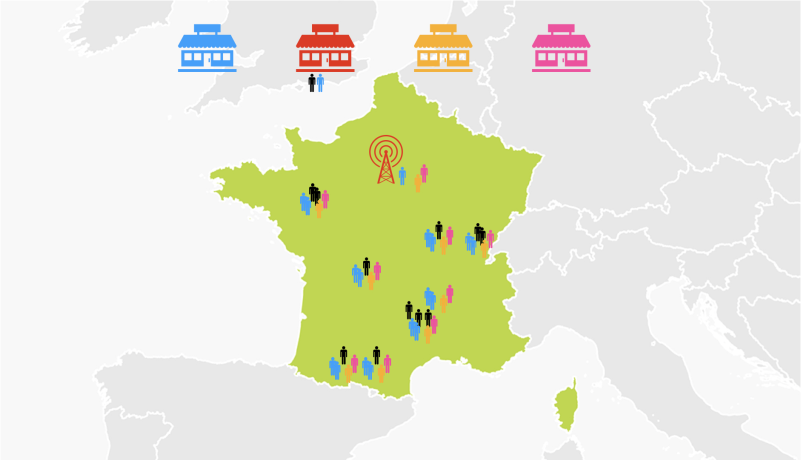

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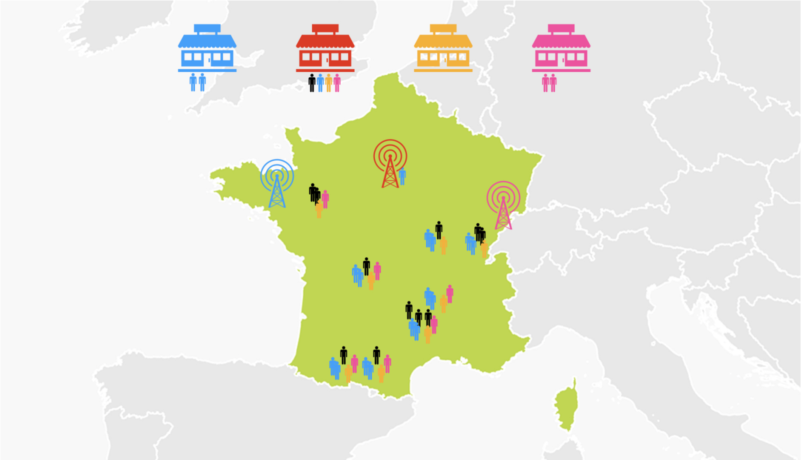

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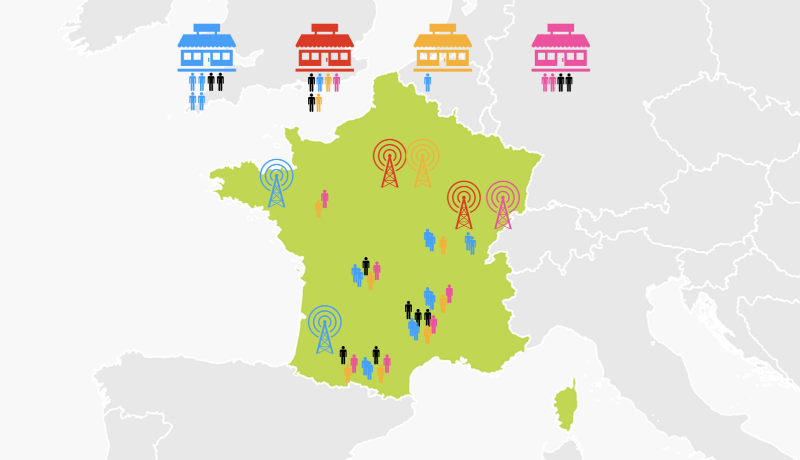


Timing game

Every scenario can be modeled through a timing game.

Parameters:

$\mathcal{I} = \{1, \ldots, N\}$, set of players;

$\mathcal{A} = \{1, \ldots, A\}$, set of sites;

$\mathcal{T} = \{1, \ldots, T\}$, set of time-intervals over which operators act to install the new technology.

$u_i : \mathcal{T}^{N \cdot A} \to \mathbb{R}$, utility function for every player i.

Variables:

$t_{ia} \in \mathcal{T}$, time at which player $i \in \mathcal{I}$ installs the technology on site $a \in \mathcal{A}$.

Timing game

Every scenario can be modeled through a timing game.

Parameters:

Z_i, maximum number of sites to be served in a time-interval by player i.

R_t, minimum number of sites to be served before time t.

Variables:

$t_{ia} \in \mathcal{T}$, time at which player $i \in \mathcal{I}$ installs the technology on site $a \in \mathcal{A}$.

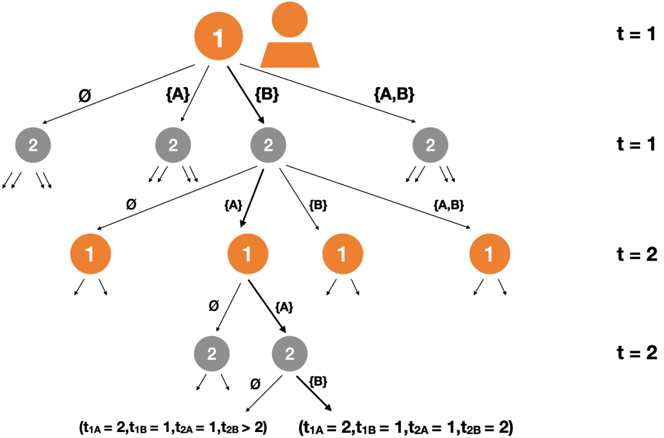
Constraints:

Logistic: $|\{a \in A, t_{ia} = t\}| \leq Z_i$ for all $i \in \mathcal{I}$ and $t \in \mathcal{T}$;

Regulatory: $|\{a \in A, t_{ia} \leq t\}| \geq R_t$ for all $i \in \mathcal{I}$;

Timing game

With $N = 2$ players, $A = 2$ sites and $T = 2$ intervals of time, the game is $T^{N \cdot A} = 16$.



Extensive-form game

Definition

An **extensive-form game** is a tuple $\Gamma = \langle \mathcal{I}, \mathcal{A}, H', H, P, u \rangle$, where:

- $\mathcal{I} = \{1, \dots, N\}$ **is the set of players**;
- H' is the set of histories with $\emptyset \in H'$;
- $\mathcal{A} : h' \in H' \to A$ is a function that provides for every history a set of actions: for all $a \in A$, we have $h' + (a) \in H'$;
- $H \subset H'$ is the set of outcomes, with the property that for all $h \in H$ we have $\mathcal{A}(h) = \emptyset$;
- $P : H' \setminus H \to \mathcal{I}$ is a function that indicates which player $P(h) \in \mathcal{I}$ acts after observing the history $h \in H' \setminus H$.
- $u = (u_i)_{i \in \mathcal{I}}$, with $u_i : H \to \mathbb{R}$, is the utility function.

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Extensive-form game

Definition

Given a game $\Gamma = \langle \mathcal{I}, \mathcal{A}, H', P, u \rangle$ and a player $i \in \mathcal{I}$, we pick all the histories at which the player acts:
$H_i = \{h \in H' \setminus H : P(h) = i\}$.
A **strategy** $s_i \in S_i$ is a function $s_i : h \in H_i \mapsto a \in \mathcal{A}(h)$ that maps every observed history $h \in H_i$ to one of the actions $a \in \mathcal{A}(h)$ available to the player.

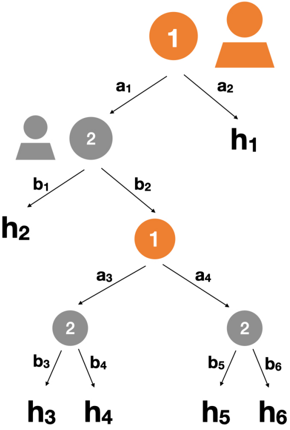
Definition

Given a game $\Gamma = \langle \mathcal{I}, H, u \rangle$, we say that a strategy profile $\langle \overline{s}_i \rangle_{i \in \mathcal{I}}$ is a **Nash equilibrium** if for every $i \in \mathcal{I}$ and for all $s_i \in S_i$:

$$u_i(\overline{s}_i, \overline{s}_{-i}) \geq u_i(s_i, \overline{s}_{-i}).$$

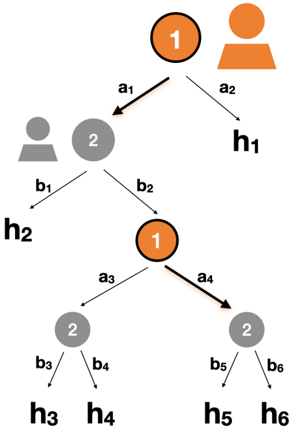
Extensive-form game

Let us analyse Nash equilibria in extensive-form games.



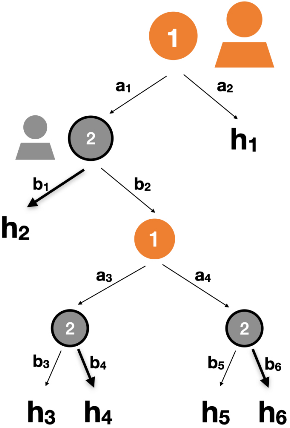
Extensive-form game

A strategy for the first player is a function that maps every node at which she plays an action.



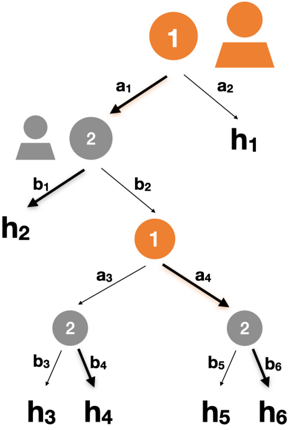
Extensive-form game

A strategy for the second player is a function that maps every node at which she plays an action.



Extensive-form game

A strategy profile leads to a unique outcome.



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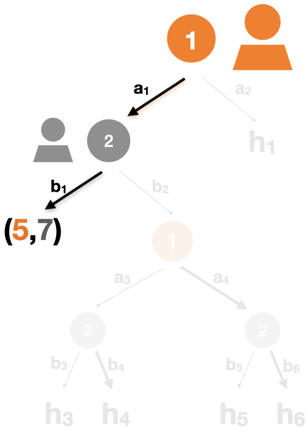
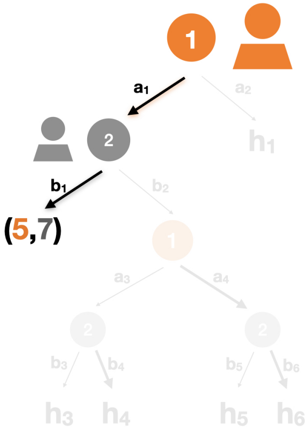


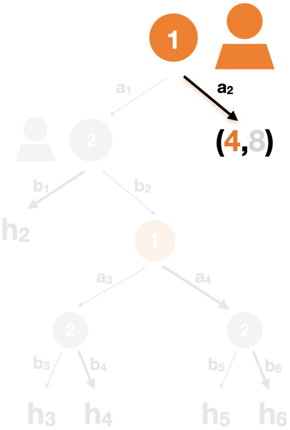
Extensive-form game

A strategy profile is a Nash equilibrium if no player has an incentive to change strategy.



Extensive-form game

A strategy profile is a Nash equilibrium if no player has an incentive to change strategy.



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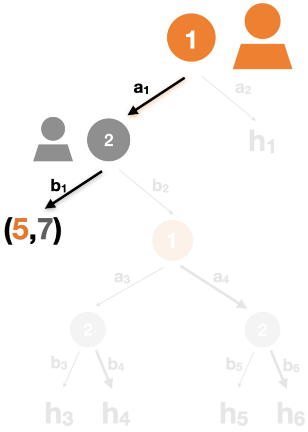

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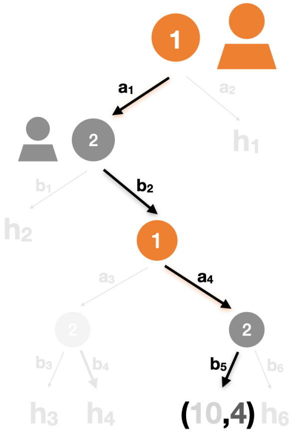
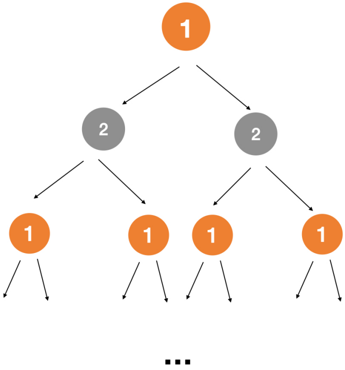


Extensive-form game

The strategies are often exponential with respect to the number of outcomes. A complete binary tree with n internal nodes has $n+1$ outcomes and 2^n strategy profiles.



Extensive-form game

The strategies are often exponential with respect to the number of outcomes.

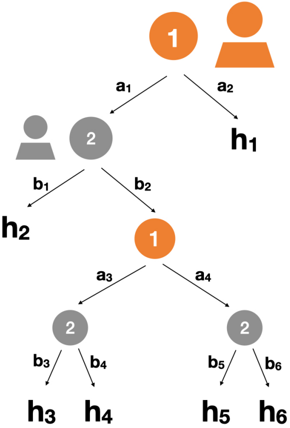
Can we enumerate the Nash equilibria of an extensive-form game, without listing all the strategies?

Definition

Given a game $\Gamma = \langle \mathcal{I}, H, u \rangle$, we say that a strategy profile $\langle \overline{s}_i \rangle_{i \in \mathcal{I}}$ is a **Nash equilibrium** if for every $i \in \mathcal{I}$ and for all $s_i \in S_i$:

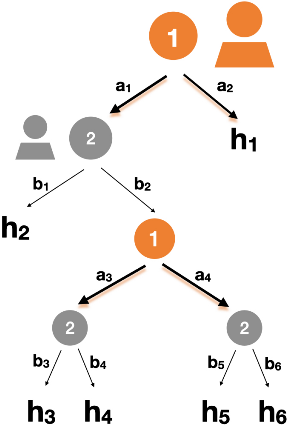
$$u_i(\overline{s}_i, \overline{s}_{-i}) \geq u_i(s_i, \overline{s}_{-i}).$$

[Von Stengel 1996]'s linear formulation is based on the concept of *sequences*.



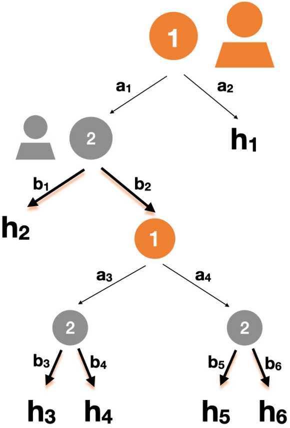
Linear formulation

The sequences of the first player are $\emptyset$, $\{a_1\}$, $\{a_2\}$, $\{a_1, a_3\}$ and $\{a_1, a_4\}$.



Linear formulation

The sequences of the second player are $\emptyset$, $\{b_1\}$, $\{b_2\}$, $\{b_2, b_3\}$, $\{b_2, b_4\}$, $\{b_2, b_5\}$ and $\{b_2, b_6\}$.



Linear formulation

Sequences can be chosen under the following constraints:

- $x_\emptyset = 1$
- $x_\emptyset = x_{\{a_1\}} + x_{\{a_2\}}$
- $x_{\{a_1\}} = x_{\{a_1, a_3\}} + x_{\{a_1, a_4\}}$

Every outcome corresponds to a pair of sequences:

- $h_1 = (\{a_2\}, \emptyset)$
- $h_2 = (\{a_1\}, \{b_1\})$
- $h_3 = (\{a_1, a_3\}, \{b_2, b_3\})$
- $h_4 = (\{a_1, a_3\}, \{b_2, b_4\})$
- $h_5 = (\{a_1, a_4\}, \{b_2, b_5\})$
- $h_6 = (\{a_1, a_4\}, \{b_2, b_6\})$

Linear formulation

[Von Stengel 1996]'s formulation can be written as a bilevel optimization problem. It is possible to obtain an equivalent linear formulation of the problem.

$$
\begin{aligned}
\max_{x} \quad & x^T U^1 \overline{y} \\
\text{s.t. } & Ex = e \\
& x \in [0,1]^{|\Lambda_1|} \\
& \overline{y} = \arg\max_{y} \quad x^T U^2 y \\
& \qquad \text{s.t.} \quad Fy = f \\
& \qquad\qquad\quad y \in [0,1]^{|\Lambda_2|}
\end{aligned}
$$

Linear formulation

Theorem

Given an extensive-form game $\langle N, H, u\rangle$, the solution $u_1^{VS} \in \mathbb{R}$ and the outcome of a Nash equilibrium $h_{NE} \in H$, we have:

$$u_1^{VS} \geq u_1(h_{NE}).$$

$$\begin{aligned} u_1^{VS} = \max_x \quad & x^T U^1 \overline{y} \\ s.t.\ & Ex = e \\ & x \in [0,1]^{|\Lambda_1|} \\ & \overline{y} = \arg\max_y \quad x^T U^2 y \\ & \qquad s.t. \quad Fy = f \\ & \qquad\qquad y \in [0,1]^{|\Lambda_2|} \end{aligned}$$

Dominated strategies

We do not compute all scenarios. We exclude those whose upper bound is too low.

Definition

A strategy is *dominated* if a player can find a different strategy that provides better utility, no matter what the other players do. If for some $s_i, s_i' \in S_i$ and for all $s_{-i} \in S_{-i}$ we have:

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i}).$$

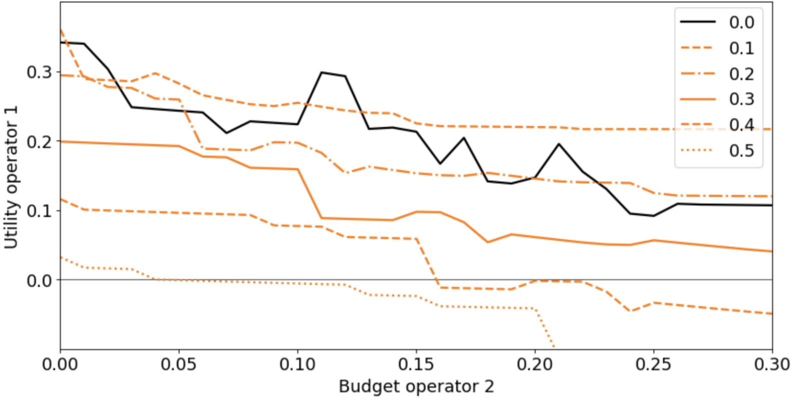
Theorem

Dominated strategies are not played at the Nash equilibrium.

Dominated strategies

A solution $u_1(h_{NE})$ is given for budget $b_1 = 0.0$.
Time for every instance: 3 minutes.

The upper bound u_1^{VS} is given for $b_1 \in \{0.1, 0.2, 0.3, 0.4, 0.5\}$.
Time for every instance: 2 seconds.



Conclusions

Results:

- Identification of operators' strategic choices for mobile network investments;
- Method to bound the utility of Nash equilibria in games.

Perspectives:

- New methods for larger instances;
- Application of the model to real-case scenarios.

Bibliography I

Bernhard Von Stengel (1996). "Efficient computation of behavior strategies". In: *Games and Economic Behavior* 14.2, pp. 220–246.

Paolo Zappalà et al. (2022). "A timing game approach for the roll-out of new mobile technologies". In: *20th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks*.